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*SOLUTION OF PROBLEM 206.*

BY R. J. ADCOCK, MONMOUTH, ILLINOIS.

EDITOR ANALYST:

Having given at pages 183-4, Vol. IV, ANALYST, a correct demonstration of the principle of Least Squares, and at pages 21-22, Vol. V, the derivation of formulæ for mean error, probable error, &c., and on pages 53-54, the solution of an example according to this corrected method, and not having made a single disciple, as appears from the published solution at p. 122, of my second ex., problem 206: I therefore propose to send you a correct solution of that example, 206, for publication in No. 5, if you will insert it, and will state my objection to Prof. S's solution.

R. J. ADCOCK.

[As Mr. Adcock has nowhere, so far as we know, explicitly pointed out the *error* which his treatment of the subject is intended to correct, we have given, above, his communication in full, and subjoin the solution and objections referred to.—Ed.]

*Solution.*—Let  $x_1, y_1, z_1, x_2, y_2, z_2, \dots, x_n, y_n, z_n$ , be the coordinates of the  $n$  points, and

$$z = cx + dy + g, \quad (1)$$

the equation of the required plane, in which the quantities  $c, d$  and  $g$  are to be determined when the plane is in its most probable position.

Now it is demonstrated at page 183-4 ANALYST, Vol. IV, that any point, line or surface, to be determined from the measured coordinates of  $n$  points, has its most probable position when the sum of the squares of the normals upon it from the  $n$  points is a minimum.

By Analytical Geometry, the normal from the point  $x_1, y_1, z_1$ , to the plane whose equation is (1) is

$$N_1 = \frac{cx_1 + dy_1 - z_1 + g}{(1 + c^2 + d^2)^{\frac{1}{2}}}. \quad (2)$$

Hence, writing  $S(N_1^2) = N_1^2 + N_2^2 + \dots + N_n^2$ , &c., and  $S(x_1 y_1)$   
 $= x_1 y_1 + x_2 y_2 + \dots + x_n y_n$ , &c.,

$$S(N_1^2) = \frac{c^2 S(x_1^2) + d^2 S(y_1^2) + S(z_1^2) + ng^2 + 2cdS(x_1 y_1)}{1 + c^2 + d^2} \\ + \frac{2cgs(x_1) - 2dS(y_1 z_1) + 2dgS(y_1) - 2cs(x_1 z_1) - 2gS(z_1)}{1 + c^2 + d^2} \quad (3)$$

Then, from (3),  $c, d$  and  $g$  are to have such values as make  $S(N_1^2)$  a minimum. Therefore

$$D_c S(N_1^2) = 0 \dots (4), \quad D_d S(N_1^2) = 0 \dots (5), \quad D_g S(N_1^2) = 0, \quad (6)$$

are the equations which determine  $c$ ,  $d$  and  $g$ ; the solution of which can be performed by equations of the second degree, by first transferring the origin of coordinates to the centre of gravity of the  $n$  points.

My objection to Prof. Scheffer's solution is, that it does not recognize the proposition in relation to normals above referred to, nor the necessary and fundamental definition of the error of a point with respect to a line or surface, which definition is, that the error is the normal from the point to the line or surface.

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#### SOLUTIONS OF PROBLEMS IN NUMBER FOUR.

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SOLUTIONS of problems in No. 4 have been received as follows:

From Prof. W. W. Beman, 215; Marcus Baker, 213, 215, 217; George C. Comstock, 215; Geo. M. Day, 212, 213, 214, 215, 216, 217, 219; Prof. A. B. Evans, 213; Geo. Eastwood, 218; Henry Gunder, 212, 213, 214, 217; W. E. Heal, 212, 213, 214; Wm. Hoover, 212, 213, 214; Henry Heaton, 212, 213, 214, 215, 216, 217, 219, 220; George H. Harvill, 213; Prof. W. W. Johson, 217, 220; Prof. J. H. Kershner, 212, 213, 214, 215, 216, 217, 219; Prof. D. J. Mc Adam, 212, 213, 214, 217; L. W. Meech, 218; Artemas Martin, 220; Prof. J. Scheffer, 212, 213, 214, 215, 216, 217; E. B. Seitz, 212, 213, 214, 215, 216, 217, 219, 220.

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211. "Prove that every number is either a triangular number or is the sum of two, or of three triangular numbers."

[No demonstration of this proposition has been received. Prof. Scheffer writes: "This theorem is a special case of a more general one. As to this case and the following one, viz.: *Every number is either a square or the sum of two, three, or four squares*, the demonstrations have been discovered, but as to the pentagonal and higher numbers, the attempts of the greatest mathematicians at a demonstration have thus far proved futile."

The writer of the article, *number*, in *Johnson's New Encyclopædia*, says, "It is a general principle, though not capable of rigorous demonstration, that any whole number is equal to the sum of 1, 2 or 3 triangular numbers, or to the sum of 1, 2, 3 or 4 square numbers, or to the sum of 1, 2, 3, 4, or 5 pentagonal numbers, etc."]

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212. "One half of a circular tract of land is cut off by an arc of a circle whose center is in the circumference of the circular tract. Find the radius with which the arc is described."